Astrapi's Spiral Polynomial Division Multiplexing (SPDM): Working Principle

Introduction: Spiral Polynomial Division Multiplexing (SPDM) exploits a new type of orthogonality in the polynomial coefficient space. The novelty of the approach lies in representing the message as a linear combination of amplitude-weighted basis functions in the polynomial coefficient space, known as Cairns functions. In another embodiment, SPDM uses the orthogonality of the basis functions to encode individual messages on decomposed sub-channels and retrieve them back at the receiver to decode the transmitted messages.

Table 1 demonstrates that the Cairns series coefficients define a set of orthogonal vectors.

Table 1: The Cairns Series Coefficients

	1	t	$\frac{t^2}{2!}$	$\frac{t^3}{3!}$	$\frac{t^4}{4!}$	$\frac{t^5}{5!}$	$\frac{t^6}{6!}$	$\frac{t^7}{7!}$	…
$\psi_{0,0}(t) = e^t$	1	1	1	1	1	1	1	1	…
$\psi_{1,0}(t) = e^{-t}$	1	-1	1	-1	1	-1	1	-1	…
$\psi_{2,0}(t) = \cos(t)$	1	0	-1	0	1	0	-1	0	…
$\psi_{2,1}(t) = \sin(t)$	0	1	0	-1	0	1	0	-1	…
$\psi_{3,0}(t)$	1	0	0	0	-1	0	0	0	…
$\psi_{3,1}(t)$	0	1	0	0	0	-1	0	0	…
$\psi_{3,2}(t)$	0	0	1	0	0	0	-1	0	…
$\psi_{3,3}(t)$	0	0	0	1	0	0	0	-1	…
…	…	…	…	…	…	…	…	…	…

The SPDM Algorithm: An overview of the key SPDM steps at the transmitter and receiver is provided below. Modification or additional signal processing may be required based on specific applications.

Transmitter

1. Each sub-channel corresponds to a Cairns function selected from $0 \leq m \leq M$, where 2^M is the number of sub-channels.
2. All sub-channels are normalized to have the same RMS power budget over the evaluation interval.
3. Synchronization and power normalization are achieved between the transmitter and receiver (see below).
4. Information is loaded onto each sub-channel through amplitude modulation of the Cairns functions based on input bit sequences.
5. The sub-channels are summed according to their amplitude modulation to produce a composite "message polynomial".
6. The message polynomial is then convolved with a "shaping polynomial" to provide smoothing and to reduce the amplitude to zero at the transmit time interval (TTI) boundaries.
7. The product of the message polynomial with the shaping polynomial is called the "transmission polynomial". The degree of the transmission polynomial is the sum of the degrees of the message and shaping polynomial.

Figure 1 shows an example of a transmission polynomial. Amplitudes generated by the transmission polynomial are shown in red: this corresponds to what is sent over the channel. Note that every TTI starts and ends at amplitude zero. The message polynomial is shown in black. The dashed black lines show the eight Cairns functions (selected from $m \leq 3$) which, when summed, produce the message polynomial. They are obtained from the message polynomial by projection onto Cairns space, and their respective amplitudes determine the sub-channel messages.

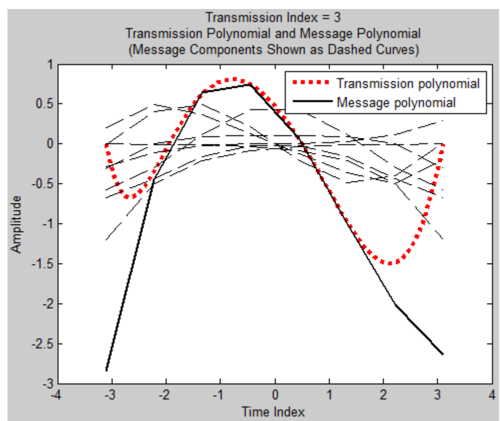

Figure 1: Example of Transmission Polynomial

Receiver

1. In the receiver, the de-multiplexer (DMX) fits a polynomial to the received data and regenerates the transmission polynomial. Up-sampling in the transmitter facilitates the regeneration of the transmission polynomial at the receiver from high-amplitude points in the middle of the transmission.
2. The DMX de-convolves the shaping polynomial from the transmission polynomial to recover the transmitted message polynomial.
3. The sub-channel amplitudes are re-constructed by projecting the message polynomial onto Cairns functions.
4. The sub-channels are converted back into bits by mapping the amplitude values for each sub-channel into bit sequences.

Synchronization and Power Control: For the receiver to correctly interpret the information the transmitter provides, the two ends of the communication channel must be synchronized to agree on when a message starts. Further, the two ends must agree on the power of the transmitted message so that amplitude information is interpreted correctly: this is referred to as “power control”. Clearly, the less power diverted from traffic the better, since the purpose of the communication channel is to convey user information. A remarkable fact is that the synchronization signature for SPDM is extremely strong, and indeed allows very accurate synchronization with no reduction in traffic for channels with relatively low impairment. During synchronization, SPDM tries to balance a very unstable function and hence a slight mismatch in phase can be detected efficiently. An analogy to the synchronization mechanism of SPDM would be balancing a pencil on its tip; a slight inclination in any direction destroys its balance.

Based on the allowable power to be transmitted for synchronization signaling, SPDM synchronization can be classified into three types:

1. ***Full Power Synchronization.*** The full channel power is made available for synchronization in a particular TTI.
2. ***Limited Power Synchronization.*** Only the channel power associated with the rising and falling exponential sub-channels is made available for synchronization in a particular TTI.
3. ***No Power Synchronization.*** No channel power (or bandwidth) is made available specifically for synchronization.

Figure 2 shows a full power correct synchronization match. Out of 8 Cairns functions ($m \leq 3$), all available power is distributed between only 2 functions, namely the rising and falling exponentials.

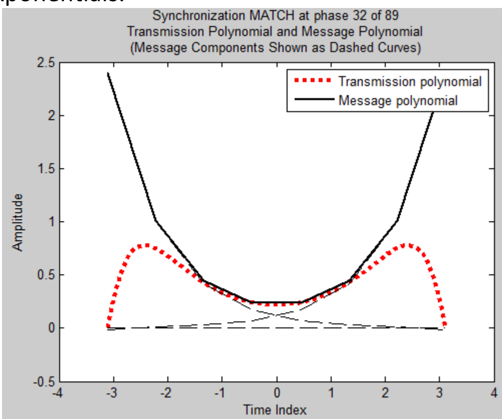

Figure 2 : Full Power Correct Synchronization Phase Match

A full power phase synchronization mismatch, where the phase offset is 1/89, is shown in Figure 3. Notice that the “mismatch” message polynomial is quite different in shape from the correctly synchronized polynomial in Figure 2, despite differing in phase by only 1 part in 89.

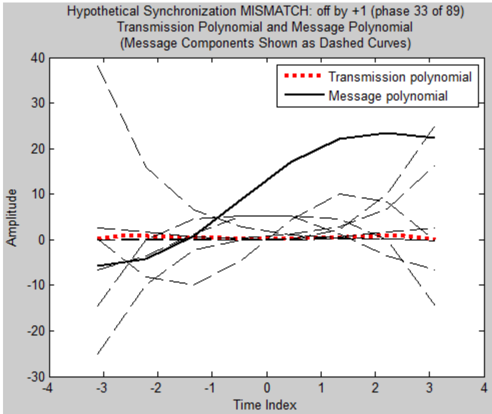

Figure 3: Full Power Synchronization Phase Match (Off by +1/89)

Figure 4 demonstrates that the synchronization signature is very strong; this is because a small displacement in phase causes either the rising or falling exponential to dominate over the other. Additionally, the gradient of the local minimums of the phase mismatch distances indicate the location of the global minimum and hence the correct phase.

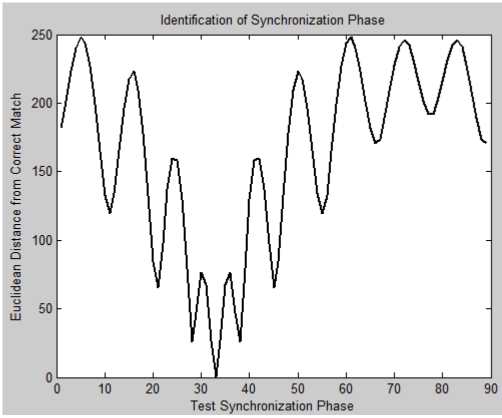

Figure 4: Full Power Synchronization Distance by Phase Match (Correct Phase is 32/89)

Conclusion: SPDM introduces a new principle for multiplexing: orthogonality in the polynomial coefficient space. The goal of SPDM is to provide a structured way to dramatically increase the flexibility of signal waveform design, and to use this additional flexibility to address key problems in telecommunications. Problems potentially addressable by SPDM include synchronization, PAPR performance, coherent interference rejection, and spectral efficiency.

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